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Letter to the Editor

Vibration control of a single-link flexible arm subjected to disturbances

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1. Introduction

Flexible manipulators, although having some inherent advantages over conventional rigid-arm robots, have more stringent requirements for the control system design, such as fast suppression of transient vibration during rapid arm movements [1]. Moreover, the vibration problem becomes more serious when the flexible arms are subjected to disturbances. A variety of control strategies for flexible manipulators have been proposed in an attempt to discover a successful and practical feedback control. Most of the previously proposed controllers have been designed by treating the control system as a deterministic nominal problem [2]. Some investigators have attempted to provide robust control logics which account for parameter variations of the flexible arms such as the change of natural frequency due to the moving mass [3–8]. However, a research on the control of flexible manipulators subjected to disturbances is rare.

An accurate end-position control of flexible manipulators is very sensitive to internal or external disturbances due to link vibrations. In this work, a new disturbance estimator using the theory of variable structure system (VSS) is proposed to enhance control performance of a single-link flexible arm subjected to torque disturbances. Various techniques for the disturbance estimation in VSS, which offer robust control performance without a prior knowledge about the disturbance, have been proposed for various control systems. Kozek et al. [9] proposed a sliding mode controller associated with linear disturbance observer and proved its effectiveness by applying it to the levitation system of high-speed electro-magnetic vehicles. Liu and Peng [10] developed a disturbance observer by treating plant non-linearities as a lumped disturbance, and showed its superior performance to the standard adaptive control scheme. Elmali and Olgac [11,12] proposed a very effective methodology, called a sliding mode control with perturbation estimation (SMCPE), which offers a robust feedback control performance, and showed its effectiveness by applying it to the SCARA robot system.

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The disturbance estimator proposed in this work has a similar form to the SMCPE. However, the proposed one does not include state derivative terms (included in the SMCPE) which may cause undesirable noise and chattering in the estimation process. Instead, the integrated average value of the imposed disturbance is used over a certain sampling period to avoid noise and chattering phenomena. After formulating the governing equation of a single-link flexible arm, a sliding mode controller with disturbance estimator is designed. The controller is then experimentally implemented and vibration control performances of the flexible arm subjected to sinusoidal torque disturbances are presented in time domain.

2. Modelling and controller design

Consider the horizontal motion of a single-link flexible manipulator as shown in Fig. 1. The uniform beam of total length l and width b is attached to the rotating hub that has a moment of inertia I_h. The axis ov' is the fixed reference line and ov is the tangential line to the beam's neutral axis at the hub. $T(t)$ is the input torque and $w'(v,t)$ the elastic deflection of the link. Upon assuming Euler–Bernoulli beam theory, small elastic deflections, small angular velocities and neglecting axial deflections, the system model can be obtained in the state space as follows [2,6]:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}T(t),\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t),\end{aligned} \tag{1}$$

where

$$\begin{aligned}\mathbf{x}(t) &= [q_0(t) \quad \dot{q}_0(t) \quad q_1(t) \quad \dot{q}_1(t) \quad \cdots \quad q_n(t) \quad \dot{q}_n(t)]^{\mathrm{T}}\\ &= [x_1(t) \quad x_2(t) \quad \cdots \quad x_{2n+2}(t)]^{\mathrm{T}},\end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & & & & & \\ 0 & 0 & & & & \mathcal{O} & \\ & & 0 & 1 & & & \\ & & -\omega_1^2 & -2\zeta_1\omega_1 & & & \\ & & & & \ddots & & \\ & \mathcal{O} & & & & 0 & 1 \\ & & & & & -\omega_n^2 & -2\zeta_n\omega_n \end{bmatrix}, \tag{2}$$

$$\mathbf{B} = \frac{1}{I_t}[0 \quad 1 \quad 0 \quad \phi_1'(0) \quad \cdots \quad 0 \quad \phi_n'(0)]^{\mathrm{T}},$$

$$\mathbf{y}(t) = [\,y_t \quad y_\theta]^{\mathrm{T}},$$

$$\mathbf{C} = \begin{bmatrix} l & 0 & \phi_1(l) & 0 & \cdots & \phi_n(l) & 0 \\ 1 & 0 & \phi_1'(l) & 0 & \cdots & \phi_n'(l) & 0 \end{bmatrix}.$$

In the above, $q_i(t)$ is the ith generalized modal co-ordinate, ϕ_i the ith eigen function, ϕ_i' the ith modal slope coefficient, I_t the total moment of inertia (hub and link). ω_i and ζ_i are the natural

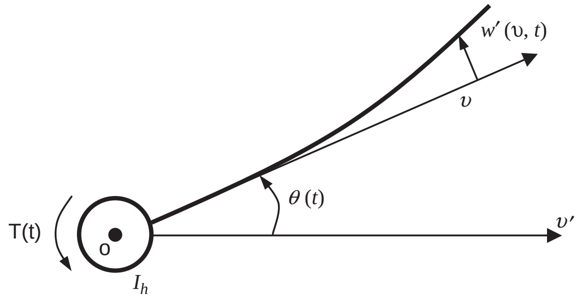


Fig. 1. A single-link flexible manipulator.

frequency and damping ratio of the ith mode, respectively. It is noted that the output matrix $\mathbf{C}$ is related to the tip position (y_t) and hub angle (y_θ).

The accuracy of the system model (1) is very important in the context of spillover problems. The accuracy depends heavily upon geometrical and material properties of the flexible arm as well as dynamic characteristics of actuators and sensors. It may be estimated by comparing transfer functions for the exact and reduced model [2] or by investigating dynamic responses before and after employing the controller [6]. In this work, the first flexible mode has been determined as a primary control mode after investigating the participation factor of each vibration mode to the total dynamic response. Thus, the control model can be written by

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t),\\
\dot{x}_2(t) &= \frac{1}{I_t}u(t) + \frac{1}{I_t}d(t),\\
\dot{x}_3(t) &= x_4(t),\\
\dot{x}_4(t) &= -\omega_1^2 x_3(t) - 2\zeta_1\omega_1 x_4(t) + \frac{\phi_1'(0)}{I_t}u(t) + \frac{\phi_1'(0)}{I_t}d(t).
\end{aligned} \tag{3}$$

In the above, $d(t)$ is external torque disturbance and $u(t)$ ($= T(t)$) is control input torque to be designed.

In order to design a sliding mode controller, we define a stable sliding surface by

$$s(t) = \sum_{i=1}^{4} c_i x_i(t), \quad c_i > 0, \tag{4}$$

where c_i is the gradient of the surface. The state $x_i(t)$ converges to zero for any initial conditions if we design an appropriate controller so as to satisfy the following sliding mode condition [13]:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}s^2(t) \leqslant -\eta|s(t)|, \quad \eta\text{: strictly positive constant.} \tag{5}$$

We first design a conventional sliding mode controller (SMC) which does not include disturbance estimator. From the sliding surface dynamic motion associated with the system model (3), the

SMC which satisfies the sliding mode condition (5) can be designed as follows:

$$
\begin{aligned}
u(t) &= u_{eq}(t) - k\,\mathrm{sgn}(s(t)), \quad k > |d| + \eta \frac{I_t}{c_2 + c_4\phi_1(0)},\\
u_{eq}(t) &= \frac{I_t}{c_2 + c_4\phi_1'(0)}(-c_1x_2(t) + c_4\omega_1^2x_3(t) + (-c_3 + 2c_4\zeta_1\omega_1)x_4(t)).
\end{aligned}
\tag{6}
$$

In the above, k is the feedback gain and $u_{eq}(t)$ is the equivalent controller without disturbance. We see from the above controller, the upper bound of the disturbance $d(t)$ should be known to guarantee robust and stable control performance. However, an accurate knowledge of the upper bound may not be easy to obtain in practice. This may yield over-conservative high feedback gains which result in undesirable control performance such as high chattering [14]. Consequently, an accurate estimation of the disturbance is necessary to enhance control performance.

In order to do this, we arrange $s(t)$ dynamics which includes control input as follows:

$$
\begin{aligned}
\dot{s}(t) = {}& c_1x_2(t) - c_4\omega_1^2x_3(t) + (c_3 - 2c_4\zeta_1\omega_1)x_4(t)\\
&+ \left(\frac{c_2 + c_4\phi_1'(0)}{I_t}\right)(u_{eq}(t) - k\,\mathrm{sgn}(s(t)) + d(t) - d_{estimated}(t)).
\end{aligned}
\tag{7}
$$

It is noted that the dynamics of $s(t)$ contains the $d_{estimated}(t)$, which is one of the input components (refer to Eq. (15)). Integration of the above equation from $T-\delta$ to T yields the following equation:

$$
\begin{aligned}
\int_{T-\delta}^{T} d(t)\,\mathrm{d}t = {}& \frac{I_t}{c_2 + c_4\phi_1'(0)}(s(T) - s(T-\delta))\\
&+ \delta\cdot k\,\mathrm{sgn}(s(T-\delta)) + \delta\cdot d_{estimated}(T-\delta)\\
&- \int_{T-\delta}^{T}\left(\frac{I_t}{c_2 + c_4\phi_1'(0)}(c_1x_2(t) - c_4\omega_1^2x_3(t)\right.\\
&\left. + (c_3 - 2c_4\zeta_1\omega_1)\,x_4(t)) + u_{eq}(T-\delta)\right)\mathrm{d}t.
\end{aligned}
\tag{8}
$$

In the above equation, δ is the sampling time (small time step) for the estimation. Both estimation and control performances, of course, depend upon the sampling time. We define a constant which is the same value as the left-hand side of Eq. (8) as follows:

$$
\int_{T-\delta}^{T} d_{average}(T)\,\mathrm{d}t = \int_{T-\delta}^{T} d(t)\,\mathrm{d}t. \tag{9}
$$

We see from the above equation that during integration time the integrated value of $d(t)$ is equal to the integrated value of $d_{average}(T)$. Thus, the average value of the disturbance is given by

$$
d_{average}(T) = \int_{T-\delta}^{T} d(t)\,\mathrm{d}t \Big/ \delta. \tag{10}
$$

Substituting Eq. (10) into Eq. (8) yields

$$\begin{aligned} d_{average}(T) = & \frac{I_t}{c_2 + c_4\phi_1'(0)} \frac{s(T) - s(T-\delta)}{\delta} \\ & + k\,\mathrm{sgn}(s(T-\delta)) + d_{estimated}(T-\delta) \\ & - \frac{1}{\delta}\int_{T-\delta}^{T}\left(\frac{I_t}{c_2 + c_4\phi_1'(0)}(c_1 x_2(t) - c_4\omega_1^2 x_3(t) \right. \\ & \left. \qquad + (c_3 - 2c_4\zeta_1\omega_1)\,x_4(t)) + u_{eq}(T-\delta)\right)\mathrm{d}t. \end{aligned} \tag{11}$$

The last term of the right-hand side in Eq. (11) is hard to calculate accurately. Thus, it is approximated by

$$X_c(T) = -(u_{eq}(T) - u_{eq}(T-\delta))/2. \tag{12}$$

The last term of Eq. (11) represents the area enclosed by actual equivalent control input $u_{eq}(t)$, while Eq. (12) is the approximated average area. Now, by substituting Eq. (12) into Eq. (11) the final form of the $d_{average}(T)$ is obtained as follows:

$$\begin{aligned} d_{average}(T) = & \frac{I_t}{c_2 + c_4\phi_1'(0)} \frac{s(T) - s(T-\delta)}{\delta} + k\,\mathrm{sgn}(s(T-\delta)) \\ & + d_{estimated}(T-\delta) - X_c(T). \end{aligned} \tag{13}$$

As defined by Eq. (9), $d_{average}(T)$ in the above equation is the value evaluated from $T-\delta$ to T. Therefore, we need to use the Taylor series in order to estimate the disturbance from T to $T+\delta$.

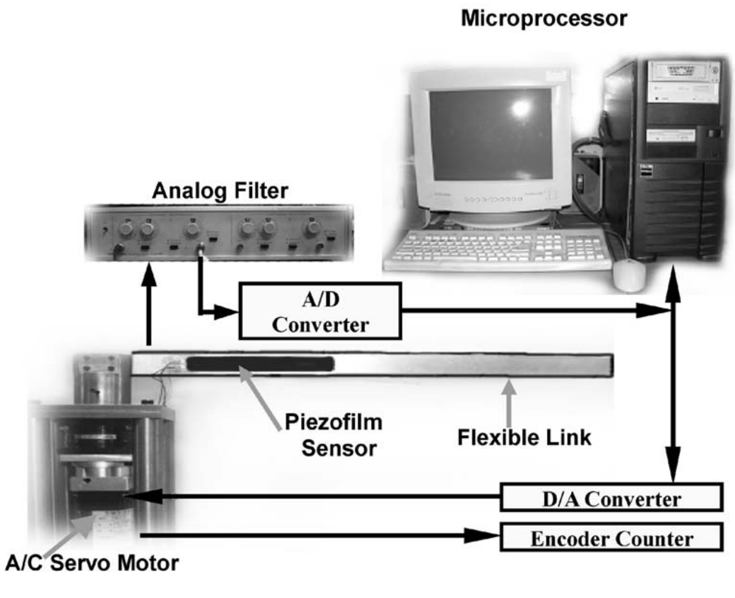


Fig. 2. Experimental apparatus for controller realization.

In other words, $d_{estimated}(t)$ can be realized by the Taylor series as follows:

$$d_{estimated}(t) = \sum_{i=0}^{n} \delta^i \cdot d^{(i)}_{average}(T)/i! \tag{14}$$

Consequently, the proposed sliding mode controller with the disturbance estimator (SMCDE) is given by

$$u(t) = u_{eq}(t) - k\,\mathrm{sgn}(s(t)) - d_{estimated}(t). \tag{15}$$

We known that, in practice, it is not desirable to use the signum function (sgn(·)) in controller (15) because of the chattering. Therefore, we may replace the signum function by the saturation function in the controller implementation [15].

Table 1
Model and control parameters of the flexible manipulator

Parameters	Value
Length of the flexible link (l)	0.5 m
Thickness of the flexible link	0.001 m
Width of the flexible link (b)	0.03 m
Mass per unit length of the link	0.7102 kg/m
Young's modulus of the link	194.6 GPa
Moment of the inertia of the hub (I_h)	0.0000692 kg m^2
First-mode damping ratio (ζ_i)	0.00518
First-mode natural frequency (ω_i)	26.06 rad/s
Control gain (k)	0.45
Sliding surface gradient (c_i)	0.8931
Estimation sampling time (δ)	0.001 s

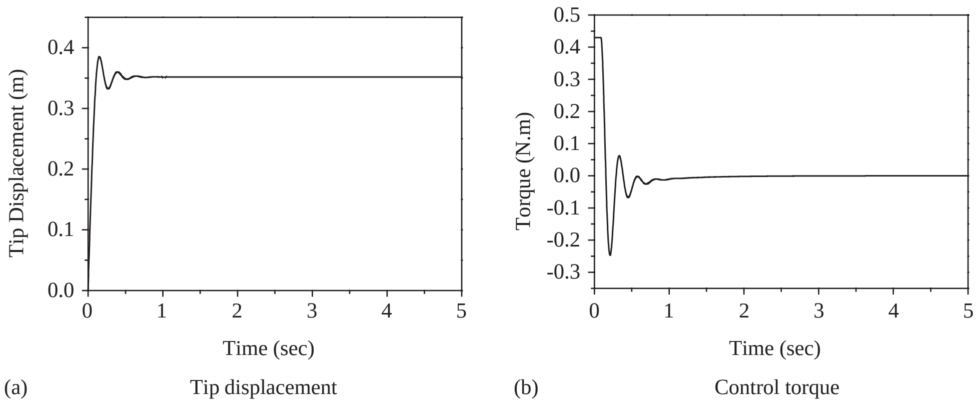
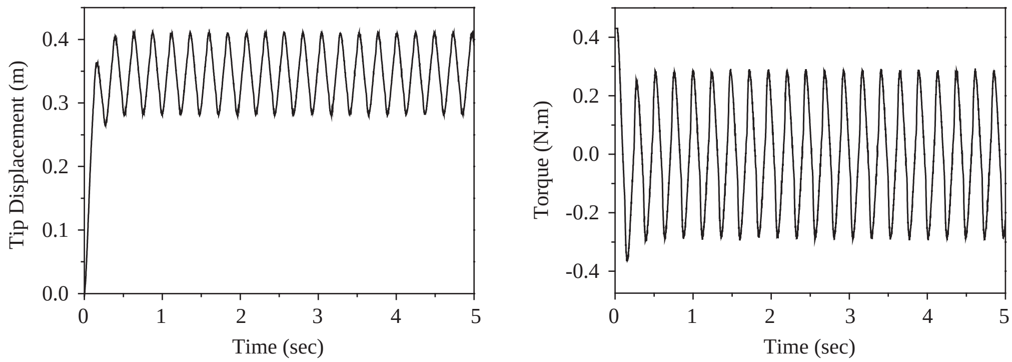
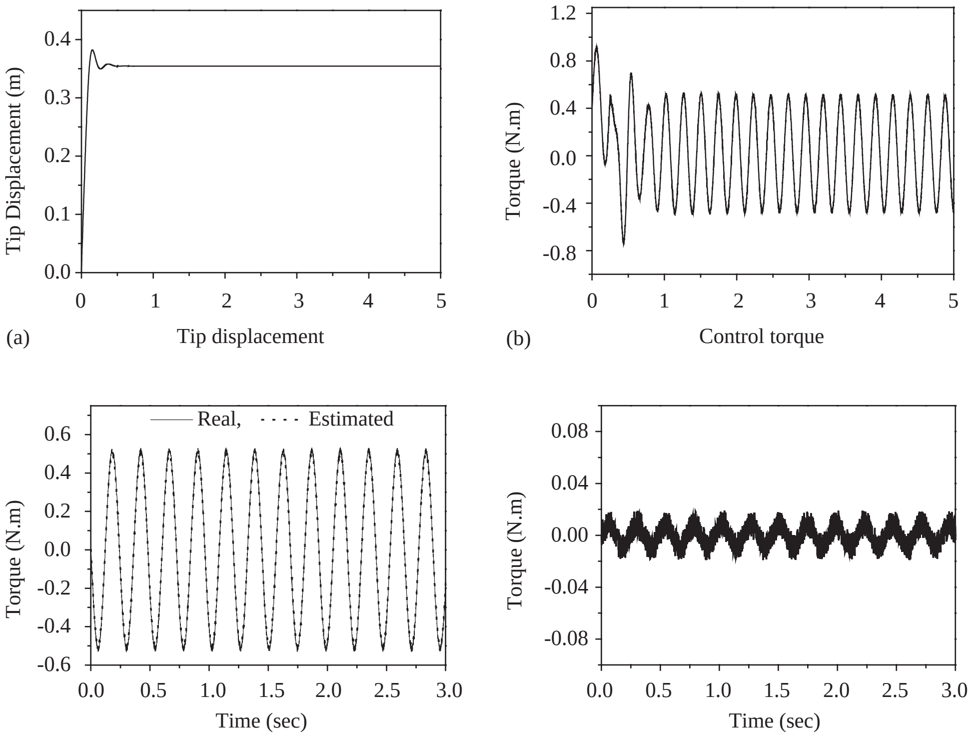


Fig. 3. Control response in the absence of torque disturbance using the SMC: (a) tip displacement, (b) control torque.



(a) Tip displacement (b) Control torque

Fig. 4. Control response in the presence of torque disturbance (4.15 Hz) using the SMC: (a) tip displacement, (b) control torque.



(a) Tip displacement (b) Control torque

(c) Disturbance estimation (d) Estimation error

Fig. 5. Control response in the presence of torque disturbance (4.15 Hz) using the SMCDE: (a) tip displacement, (b) control torque, (c) disturbance estimation, (d) estimation error.

3. Experimental results

In order to demonstrate superior control performance of the proposed method, an experimental apparatus is established as shown in Fig. 2. The angular displacement of the motor is directly

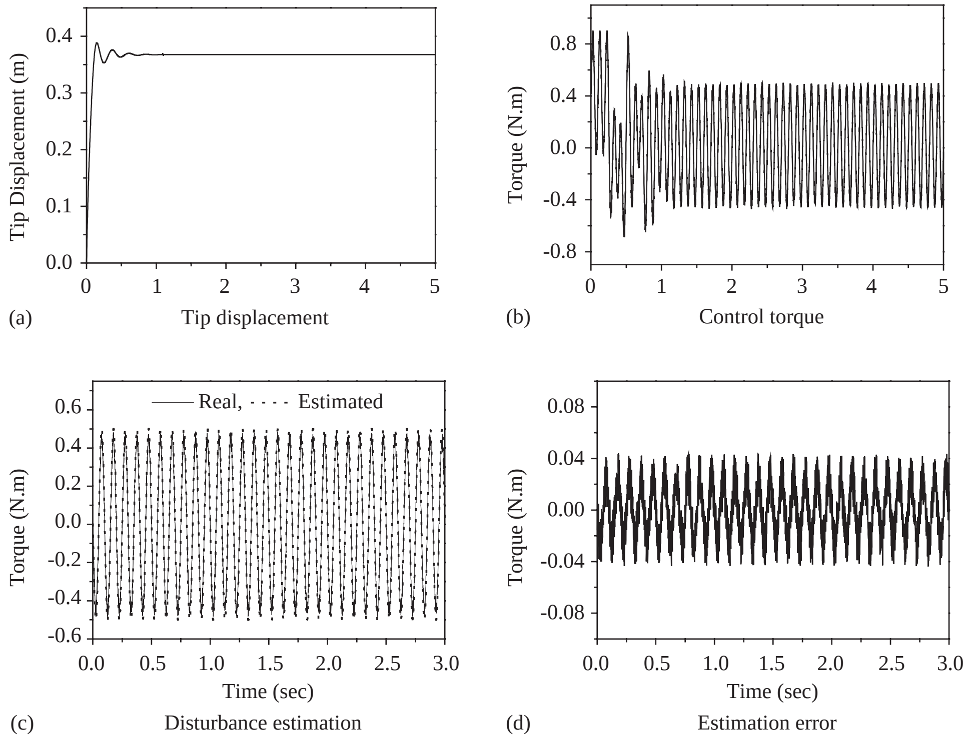


Fig. 6. Control response in the presence of torque disturbance (10 Hz) using the SMCDE: (a) tip displacement, (b) control torque, (c) disturbance estimation, (d) estimation error.

measured from the optical encoder, while the elastic deflection of the link from the piezofilm sensor. The signal from the piezofilm sensor goes to analog filter to eliminate high-frequency components of the flexible link before being fed back to the microprocessor. The geometrical and material properties of the flexible arm are presented in Table 1. The control parameters used in this experimental work are also given in Table 1.

Fig. 3 presents the measured control response in the absence of the torque disturbance. We clearly see that the tip displacement of the flexible manipulator is well settled without exhibiting undesirable vibration. This response has been obtained by implementing the sliding mode controller (SMC) given by Eq. (6). Fig. 4 presents the measured control response in the presence of the torque disturbance; $0.2\sin(2\pi \times 4.15 \times t)$ Nm. It is noted that the frequency of the torque disturbance (4.15 Hz) is same as the first-mode natural frequency of the flexible arm. As expected, we have undesirable large vibration magnitude when we use the SMC only. This directly implies the necessity of the disturbance estimator to achieve favorable control performance.

Fig. 5 presents control responses of the flexible manipulator subjected to the torque disturbance; $0.5\sin(2\pi \times 4.15 \times t)$ Nm. In this case, the sliding mode controller with disturbance estimator (SMCDE) proposed in this work has been implemented. We clearly see that the tip displacement of the flexible arm is well settled to the desired position without showing vibration problem. It is also seen from Figs. 5(c) and (d), the imposed torque disturbance has been well estimated by the proposed methodology. Favorable control response has been obtained even if we change the disturbance frequency from 4.15 to 10 Hz as shown in Fig. 6. It is noted that the estimation error increases as the disturbance frequency is increased. This is, of course, directly related to the estimation sampling time.

4. Concluding remarks

A new type of a sling mode controller with disturbance estimator has been proposed for vibration control of a flexible-link manipulator. The proposed estimator is featured by an integrated average value of the imposed disturbance over a certain sampling time. In addition, the proposed estimator does not include a time derivative of higher order state variables which may cause undesirable chattering. It has been demonstrated through experimental implementation that the proposed control methodology can offer accurate estimation of the imposed disturbance, and hence provide superior control performance of the system subjected to external disturbance. The control results presented in this work are quite self-explanatory justifying that the sliding mode controller with disturbance estimator can be effectively applied for successful vibration control of flexible structures systems subjected to disturbances. It is finally remarked that an efficient estimation method for various types of disturbances such as step disturbance is being developed.

References

[1] W.J. Book, M. Majette, Controller design for flexible, distributed parameter mechanical arms via combined state space and frequency domain techniques, *Transactions of the American Society of Mechanical Engineers, Journal of Dynamic Systems, Measurement and Control* 105 (1983) 245–254.

[2] E. Schmitz, Experiments on the End-point Control of Very Flexible One-link Manipulator, Ph.D. Thesis, Department of Aeronautics and Astronautics, Stanford University, 1985.

[3] B.S. Yuan, W.J. Book, A robust scheme for direct adaptive control of flexible arms, *Proceedings of the American Society of Mechanical Engineers, Winter Annual Meeting* 6 (1987) 261–268.

[4] S.B. Choi, C.C. Cheong, B.S. Thompson, M.V. Gandhi, Vibration control robust to parameter variations in a very flexible manipulator fabricated from composite laminates, *Journal of Sound and Vibration* 162 (1993) 195–208.

[5] K.S. Yeung, Y.P. Chen, Sliding mode controller design of a single-link flexible manipulator under gravity, *International Journal of Control* 52 (1990) 101–117.

[6] S.B. Choi, C.C. Cheong, H.C. Shin, Sliding mode control of vibration in a single-link flexible arm with parameter variations, *Journal of Sound and Vibration* 179 (1995) 737–748.

[7] J.H. Ryu, D.S. Kwon, Y. Park, A robust controller design method for flexible manipulator with large time varying payloads and parameter uncertainties, *Journal of Intelligent and Robotic Systems* 27 (2000) 345–361.

[8] R.J. Theodore, A. Ghosal, Robust control of multilink flexible manipulator, *Mechanism and Machine Theory* 38 (4) (2003) 367–377.

[9] M. Kozek, G. Puchhammer, M.P. Jorgl, A MIMO sliding mode control for a high speed EMSMAGLEV-vehicle using disturbance estimation, *Proceedings of the IEEE International Conference on Robotics and Automation*, 1994, pp. 587–592.

[10] C.S. Liu, H. Peng, Disturbance observer based tracking control, *American Society of Mechanical Engineers, Journal of Dynamic Systems, Measurement and Control* 122 (2000) 332–335.

[11] H. Elmali, N. Olgac, Sliding mode control with perturbation estimation (SMCPE): a new approach, *International Journal of Control* 56 (1992) 923–941.

[12] H. Elmali, N. Olgac, Implementation of sliding mode control with perturbation estimation (SMCPE), *IEEE Transactions on Control System Technology* 4 (1992) 79–85.

[13] J.J. Slotine, S.S. Sastry, Tracking control of non-linear systems using sliding surfaces with application to robot manipulators, *International Journal of Control* 38 (1983) 465–492.
[14] V.I. Utkin, Variable structure systems with sliding modes, *IEEE Transactions on Automatic Control* 22 (1977) 212–222.
[15] S.B. Choi, C.C. Cheong, D.W. Park, Moving switching surfaces for robust control of second-order variable structure systems, *International Journal of Control* 58 (1993) 229–245.